ITG modes

Some characteristics :

- ignore trapped particles

  $\omega \ll \omega_b$, i.e

- $k_{\parallel} v_{Te} \gg \omega \gg k_{\parallel} v_{Ti}$ ions are moving slowly along field

  e^- moving rapidly $\Rightarrow$

  $g_e = 0$

- also assume small drift

  $\omega \gg \omega_{di}$

$$\overset{0}{\cancel{v_{\parallel} \nabla_{\parallel} g_i}} - i(\omega - \omega_{d_i}) g_i = -\frac{i e_i}{T_i} J_0 \phi (\omega - \omega_{*i}^{T}) f_{i0}$$

$$g_i = \frac{e_i \phi}{T_i} \frac{(\omega - \omega_{*i}^{T})}{(\omega - \omega_{di})} J_0 f_{i0}$$

quasi-neutrality

$$\sum_a' \frac{n_a e_a^2}{T_a}\phi = \underbrace{\sum_a' e_a \int g_a J_0 \, d^3v}_{\text{only ions}}$$

$n_e = n_i$

$T_e = T_i$

$\cancel{\left(\tau = T_e / T_i\right)}$

$$\frac{2ne^2}{T}\phi = \frac{e^2\phi}{T}\int \frac{\omega - \omega_{*i}^T}{\omega - \omega_{di}} J_0 f_{i0} \, d^3v$$

now $\omega_{di} \ll \omega$ $\quad \frac{1}{\omega - \omega_{di}} \simeq \frac{1}{\omega}\left(1 - \frac{\omega_{di}}{\omega}\right)$

$$= \frac{e^2\phi}{T}\int \frac{(\omega - \omega_{*i}^T)}{\omega}\left(1 - \frac{\omega_{di}}{\omega}\right) J_0\left(\frac{k_\perp v_\perp}{\Omega_a}\right) f_{i0} \, d^3v$$

v^2

$$\omega_{*i}^T = \omega_{*i}\left(1 + \eta\left(\frac{E}{T} - \frac{3}{2}\right)\right)$$

$E \propto v^2$

$\propto (v_\parallel^2 + v_\perp^2)$

$$\omega_{di} \simeq \tilde{\omega}_{di}\left[\frac{v_\perp^2}{2} + v_\parallel^2\right]$$

2π

- $d^3v = d\theta \, v_\perp \, dv_\perp \, dv_\parallel$
- normalised velocity to thermal velocity v_T

$$x_\parallel = \frac{v_\parallel}{v_T}, \quad x_\perp = \frac{v_\perp}{v_T}, \quad v_T = \sqrt{\frac{2T}{m}}$$

Trapped-electron modes
(trapped-particle modes)

- important characteristics
  - $g_e \neq 0$
  - $\omega_{be} \gg \omega$

again, GK equation

$$\underbrace{v_\parallel \nabla_\parallel g_a}_{\text{dominant}} - i(\omega - \omega_{da}) g_a = -\frac{i e_a}{T_a} J_0 \phi (\omega - \omega_{*a}^T) f_{a0}$$

expand $g_a = g_{a0} + g_{a1} + \dots \quad g_{a1} \ll g_{a0}$

1st lowest order $\quad v_\parallel \nabla_\parallel g_{a0} = 0 \quad g_{a0} = f(\ell)$

along the field ("z")

next order

$$v_\parallel \nabla_\parallel g_{a1} - i(\omega - \omega_{da}) g_{a0} = -\frac{i e_a}{T_a} J_0 \phi (\omega - \omega_{*a}^T) f_{a0}$$

now, take field line average

$$\overline{(\ldots)} = \frac{\oint \ldots \frac{dl}{v_\parallel}}{\tau_b \left\{ \oint \frac{dl}{v_\parallel} \right.}$$ time average

first term $\oint v_\parallel D_\parallel g_{a_1} \frac{dl}{|v_\parallel|} = \sigma \oint D_\parallel g_{a_1} \, dl \overset{!}{=} 0$

because $g_{a_0} \neq f(l)$

$$-i(\omega - \overline{\omega_{da}}) g_{a_0} = -\frac{ie_a}{T_a} \overline{J_0 \phi} (\omega - \omega_{*a}{}^{T}) f_{a0}$$

$$g_{a_0} = \frac{e_a}{T_a} \frac{\omega - \omega_{*a}{}^{T}}{\omega - \overline{\omega_{da}}} \overline{J_0 \phi} f_{a0}$$

$J_0 \approx 1$ always for e^-